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FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF ORTHOTROPIC PLATES OF REGULAR POLYGONAL SHAPE CARRYING A CONCENTRATED MASS

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1. Introduction

The present study deals with the solution of the title problem using a conformal mapping approach coupled with the optimized Rayleigh–Ritz method [1, 2]. By conformally transforming the given shape in the z-plane onto a unit circle in the ζ-plane it is possible to construct co-ordinate functions which satisfy the essential boundary conditions in the case of simply supported and clamped plates. For the sake of simplicity, the azimuthal variation in the ζ-plane is disregarded and the following co-ordinate functions are used.

(1) Simply supported plates:

$$W \simeq W_a = C_1(1 - r^p) + C_a(1 - r^{p+1}) + C_2(1 - r^{p+2}), \qquad \zeta = r\, e^{i\Theta}. \tag{1}$$

(2) Clamped plates:

$$W \simeq W_a = C_1(1 - r^p)^2 + C_2(1 - r^{p+1})^2 + C_3(1 - r^{p+2})^2 \tag{2}$$

where p is Rayleigh's optimization parameter [2].

It should be pointed out that orthotropic plates are commonly used in engineering practice, e.g., printed circuit boards used in electronics applications.

2. Approximate solution

Following Lekhnitskii's standard notation [3] one expresses the governing functional in the form

$$J(W) = \iint_p (D_1 W_{x^2}^2 + 2D_1 \nu_2 W_{x^2} W_{y^2} + D_2 W_{y^2}^2 + 4D_k W_{xy}^2)\, dx\, dy - \rho h \omega^2$$

$$\iint_p W^2\, dx\, dy - M\omega^2 W^2(0, 0), \tag{3}$$

where it has been assumed that the concentrated mass M is rigidly attached at the center of the plate.

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In the case of regular polygons of degree "s" the mapping function is given by [1]

$$z = A_s a_p F(\zeta) = A_S a_p \int_0^{\zeta} \frac{\mathrm{d}\zeta}{(1+\zeta^s)^{2/s}}, \tag{4}$$

where a_p is the apothem of the polygon.

Defining now

$$U_1 + V_1 i = \frac{1}{4}\frac{\mathrm{e}^{-2\Theta i}}{F'^2(\zeta)}, \qquad U_2 + V_2 i = \frac{1}{2}\frac{F''(\zeta)}{F'^3(\zeta)}\mathrm{e}^{-\Theta i}, \tag{5}$$

and substituting equations (4) and (5) into equation (3) one obtains the transformed energy functional in the form

$$\frac{A_s^2 a_p^2}{D_1} J(W_a) = \iint_c \left\{ \left[2\left(\left(W_a - \frac{W_{ar}}{r}\right)U_1 - W_{ar} U_2\right) + \frac{1}{2}\frac{W_{ar^2} + \dfrac{W_{ar}}{r}}{|F'(\zeta)|^2} \right]^2 \right.$$

$$- 8\nu_2 \left[\left(W_{ar^2} - \frac{W_{ar}}{r}\right)U_1 - W_{ar} U_2\right]^2 + \frac{\nu_2}{2}\frac{\left(W_{ar^2} + \dfrac{W_{ar}}{r}\right)^2}{|F'(\zeta)|^4}$$

$$+ \frac{D_2}{D_1}\left[-2\left(\left(W_{ar^2} - \frac{W_{ar}}{r}\right)U_1 - W_{ar} U_2\right) + \frac{1}{2}\frac{W_{ar^2} + \dfrac{W_{ar}}{r}}{|F'(\zeta)|^2} \right]^2$$

$$\left. + 16\frac{D_k}{D_1}\left[\left(W_{ar^2} - \frac{W_{ar}}{r}\right)V_1 - W_{ar} V_2\right]^2 \right\} |F'(\zeta)|^2 r \,\mathrm{d}r\,\mathrm{d}\theta$$

$$- \frac{A_s^2}{16 tg^4 \dfrac{\pi}{s}} \Omega^2 \left[A_s^2 \iint_c W_r^2 |F'(\zeta)|_r^2 \,\mathrm{d}r\,\mathrm{d}\theta + \mu s tg \frac{\pi}{s} W_{(0)}^2 \right], \tag{6}$$

where the fact that W_a, defined in equations (1) and (2), does not contain the azimuthal variable θ has been taken into account, and $\mu = M/M_p$, $M_p =$ plate mass, $tg\pi/s = \tan \pi/s$, and $\Omega_1^2 = (\rho h a^4/D_1)\omega_1^2$.

3. NUMERICAL RESULTS

Tables 1 and 2 depict values of the fundamental frequency coefficient Ω_1 in the case of isotropic, simply supported and clamped plates, respectively. Reasonably

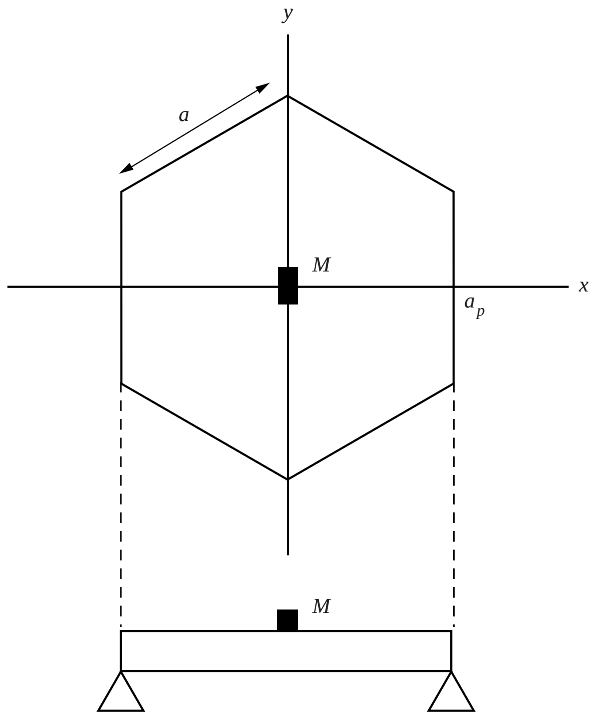

Figure 1. Orthotropic plate of regular polygonal shape carrying a central, concentrated mass, M.

good agreement with values available in the literature, for the case where $\mu = 0$, is obtained. Pentagonal, exagonal and heptagonal plates are considered in the present investigation.

TABLE 1

Frequency coefficients of simply supported isotropic plates of regular polygonal shape ($\nu = 0{\cdot}30$)

s	$\mu = 0$	0·10	0·20	0·30	0·40	$\mu = 0$ [4]
5	11·00	9·28	8·14	7·33	6·72	11·01
6	6·96	5·90	5·19	4·68	4·30	7·15
7	4·97	4·22	3·72	3·35	3·08	5·06

TABLE 2

Frequency coefficients of clamped isotropic plates of regular polygonal shape

s	$\mu = 0$	0·10	0·20	0·30	0·40	$\mu = 0$ [5]
5	19·24	15·11	12·76	11·23	10·13	19·71
6	12·56	9·91	8·39	7·39	6·68	12·81
7	8·91	7·05	5·97	5·27	4·76	9·08

TABLE 3

Frequency coefficients of simply supported orthotropic plates of regular polygonal shape ($\nu_2 = 0{\cdot}30$)

D_2/D_1	D_k/D_1	s	$\mu = 0$	0·10	0·20	0·30	0·40	$\mu = 0$ [6]
4	0·85	5	17·08	14·41	12·64	11·38	10·42	17·03
		6	10·73	9·10	8·01	7·23	6·63	11·70
		7	7·68	6·51	5·73	5·17	4·75	–
1	0·85	5	12·49	10·54	9·24	8·32	7·62	12·54
		6	7·78	6·60	5·81	5·24	4·81	8·48
		7	5·50	4·67	4·12	3·71	3·41	–
1	0·10	5	10·17	8·58	7·53	6·78	6·21	10·45
		6	6·50	5·51	4·85	4·37	4·01	7·16
		7	4·68	3·97	3·49	3·15	2·89	–
0·25	0·10	5	8·33	7·04	6·17	5·56	5·10	8·65
		6	5·35	4·54	3·99	3·60	3·30	5·94
		7	3·84	3·26	2·87	2·59	2·37	–

Tables 3 and 4 present values of Ω_1 for simply supported and clamped orthotropic plates. The following orthotropic parameters have been considered: $D_2/D_1 = 4$, $D_k/D_1 = 0{\cdot}85$, $\nu_2 = 0{\cdot}30$; $D_2/D_1 = 1$, $D_k/D_1 = 0{\cdot}85$, $\nu_2 = 0{\cdot}30$; $D_2/D_1 = 1$, $D_k/D_1 = 0{\cdot}10$, $\nu_2 = 0{\cdot}30$; $D_2/D_1 = 0{\cdot}25$, $D_k/D_1 = 0{\cdot}10$, $\nu_2 = 0{\cdot}30$.

In the case of bare plates ($\mu = 0$) the eigenvalues have been compared with those determined in reference [6].

TABLE 4

Frequency coefficients of clamped orthotropic plates of regular polygonal shape ($\nu_2 = 0{\cdot}30$)

D_2/D_1	D_k/D_1	s	$\mu = 0$	0·10	0·20	0·30	0·40	$\mu = 0$ [6]
4	0·85	5	29·77	23·37	19·73	17·36	15·67	30·46
		6	19·42	15·31	12·96	11·41	10·31	20·18
		7	13·79	10·90	9·23	8·14	7·36	14·82
1	0·85	5	21·69	17·02	14·36	12·63	11·40	22·09
		6	14·13	11·14	9·43	8·30	7·50	14·52
		7	9·99	7·90	6·69	5·90	5·33	10·62
1	0·10	5	17·90	14·06	11·88	10·45	9·44	18·66
		6	11·70	9·23	7·82	6·89	6·22	12·25
		7	8·32	6·58	5·58	4·92	4·45	9·05
0·25	0·10	5	14·71	11·56	9·77	8·59	7·76	15·38
		6	9·62	7·59	6·43	5·67	5·12	10·18
		7	6·83	5·41	4·58	4·04	3·65	7·41

Present results are, in general, somewhat lower. Possibly the values determined in reference [6] are rather high upper bounds since a single approximating function was used.

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